CCHS Mathematics A-Level Bridging Work

Answer **all** of the following questions on material from GCSE Mathematics. Check your answers using the answers provided at the end of the document. There will be a test on this material in September.

Expanding brackets (Review Ex. 3)

Q1 Remove the brackets and simplify the following expressions:

a) $(a + b)(a - b)$ b) $(p + q)(p + q)$ c) $35xy + 25y(5y + 7x) - 100y^2$

d) $(x + 3y + 2)(3x + y + 7)$ e) $(c + 2d)(c - d)(2d - 3c)$ f) $[(s - 2t)(s + 2t)]^2$

Q2 Expand and simplify:

a) $(2x^2 + 7x - 8)(6x^2 - 5x - 10)$ b) $3p(q - 5r) + 4q(p + 2r) - 7r(2p - 3q)$

Q3 The length of the longest side of a cuboid-shaped box is x cm. The shortest side of the box is 10 cm shorter than the longest side, and the other side is 8 cm shorter than the longest side. Find an expression for the total volume of 5 of these boxes, in terms of x.
Your final answer should contain no brackets.

Q4 Given that $p(x) = 3x^3 - 7x + 5$ and $q(x) = x^2 - 3x + 1$,
and $p(x) - (2 - 3x)q(x) \equiv ax^3 + bx^2 + cx + d$, find the values of a, b, c and d. *[3 marks]*

Algebraic Fractions (Ex. 3.3)

Q2 Express the following as single fractions in their simplest form.

a) $\dfrac{5}{y-1} + \dfrac{3}{y-2}$ b) $\dfrac{7}{r-5} - \dfrac{4}{r+3}$ c) $\dfrac{8}{p} - \dfrac{1}{p-3}$

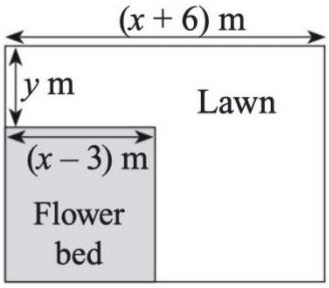
d) $\dfrac{w}{2(w-2)} + \dfrac{3w}{w-7}$ e) $\dfrac{z+1}{z+2} - \dfrac{z+3}{z+4}$ f) $\dfrac{1}{q+1} + \dfrac{3}{q-2}$

g) $\dfrac{x}{x+z} + \dfrac{2z}{x-z}$ h) $\dfrac{y}{2x+3} - \dfrac{2y}{3-x}$ i) $\dfrac{5}{r-4} + \dfrac{3}{r} - \dfrac{r}{r+1}$

Review Ex. 3

Q15 The diagram on the right shows part of a garden.
The combined area of the lawn and flower bed is $3x^2$ m^2.
The area of the flower bed is x^2 m^2.
Show that y can be expressed as:

$$y = \frac{x^2(2x - 15)}{(x+6)(x-3)}$$



Indices (Review Ex. 3)

Q16 Simplify:

a) $x^3 \times x^5$ b) $a^7 \times a^8$ c) $\frac{x^8}{x^2}$

d) $(a^2)^4$ e) $(xy^2)(x^3yz)$ f) $\frac{a^2b^4c^6}{a^3b^2c}$

Q17 Simplify:

a) $g^2 \times g^{-5}$ b) $p^4r^2 \div p^5r^{-6}$ c) $\left(k^{\frac{1}{3}}\right)^6$

d) $(mn^8 \times m^4n^{-11})^{-2}$ e) $s^4t^3 \times \left(\frac{1}{s^2t^5}\right)^{-3}$ f) $\frac{a^2}{b^2c} \times \frac{b^6}{a^4c^{-2}} \div \frac{c^2}{a^3b}$

Q18 Fully simplify $\frac{(3a^2b^2)^2 \times (2a^2b)^2}{(8a^6b^{-3})^{\frac{1}{3}}}$ *[2 marks]*

Q19 Find x such that:

a) $9^x = 3$ *[1 mark]*

b) $9^{3x} \cdot 81^{2x-1} = 27$ *[3 marks]*

Q20 Work out the following:

a) $16^{\frac{1}{2}}$ b) $8^{\frac{1}{3}}$ c) $81^{\frac{3}{4}}$

d) x^0 e) $49^{-\frac{1}{2}}$ f) $\frac{1}{27^{-\frac{2}{3}}}$

Surds (Review Ex. 3)

Q24 Simplify:

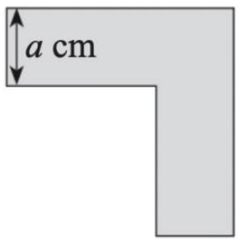
a) $\sqrt{28}$ b) $\sqrt{\frac{5}{36}}$ c) $\sqrt{18}$ d) $\sqrt{\frac{9}{16}}$

Q25 Simplify the following expressions by writing them in the form $k\sqrt{x}$, where k and x are integers and x is as small as possible.

a) $\sqrt{3} - \sqrt{12}$ b) $3\sqrt{5} + \sqrt{45}$ c) $\sqrt{7} + \sqrt{448}$

d) $\sqrt{52} + \sqrt{117}$ e) $4\sqrt{150} + \sqrt{54} - \sqrt{5}\sqrt{120}$

Q26 The diagram on the right shows a shape which has been made by cutting a small square from one corner of a larger square. The area of the larger square was 1920 cm². The area of the smaller square was 1080 cm². Find the value of a. Give your answer in the form $k\sqrt{x}$, where k and x are integers and x is as small as possible.



Q30 Express $(3\sqrt{5}-5\sqrt{3})^2$ in the form $a(b+\sqrt{c})$ where a, b and c are integers. *[3 marks]*

Q31 Show that: a) $\frac{8}{\sqrt{2}} = 4\sqrt{2}$ b) $\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

Q32 Rationalise the denominator of $\frac{2}{3+\sqrt{7}}$.

Q33 a) Write $(2\sqrt{2}+\sqrt{3})^2$ in the form $a+b\sqrt{6}$ where a and b are integers. *[2 marks]*

b) Hence, or otherwise, rationalise the denominator of this expression:

$$\frac{1}{(2\sqrt{2}+\sqrt{3})^2}$$ *[2 marks]*

Q34 Write the following in the form $p+q\sqrt{r}$, where r is an integer, and p and q are integers or fractions:

a) $\frac{11+\sqrt{13}}{5-\sqrt{13}}$ b) $\frac{2\sqrt{7}+9}{3-\sqrt{7}}$ c) $\frac{3\sqrt{5}+\sqrt{15}}{\sqrt{60}-\sqrt{20}}$

Q35 Fully simplify the number given by this expression:

$$\frac{(\sqrt{32}+\sqrt{128})^2}{\sqrt{5}+\sqrt{3}}$$ *[3 marks]*

Factorising (Ex. 4.1)

Q1 Factorise the following expressions.

a) x^2-6x+5 b) $x^2-3x-18$
c) $x^2+22x+121$ d) x^2-12x
e) $y^2-13y+42$ f) $x^2+51x+144$
g) x^2-121 h) $x^2-35x+66$

Q1 Hint If b or c is zero, use the factorising methods from Chapter 3. For d), you can't just divide by x — you would miss the solution $x = 0$ when solving.

Q2 Solve the following equations.

a) $x^2-3x-10=0$ b) $2x^2+2x-40=0$
c) $p^2+21p+38=0$ d) $x^2-15x+54=0$
e) $x^2+18x=-65$ f) $x^2-x=42$
g) $x^2+1100x+100\,000=0$ h) $3x^2-3x-6=0$

Problem Solving

Look out for questions where the equation can be simplified before factorising — for example by dividing through by a number.

Q3 Factorise the following expressions.

a) $4x^2-4x-3$ b) $2x^2+23x+11$ c) $7x^2-19x-6$ d) $-x^2-5x+36$
e) $6x^2-7x-3$ f) $2x^2-2$ g) $3x^2-3$ h) $-x^2+9x-14$

Quadratic Formula

Exercise 4.2

Q1 Solve the following equations using the quadratic formula, giving your answers in surd form where necessary.

a) $x^2 - 4x = -2$
b) $x^2 - 2x - 44 = 0$
c) $x^2 + 3x = 12$
d) $x^2 - 14x + 42 = 0$
e) $4x^2 + 4x - 1 = 0$
f) $-x^2 + 4x - 3 = 0$
g) $x^2 - \frac{5}{6}x + \frac{1}{6} = 0$
h) $x^2 - 2\sqrt{11}x + 11 = 0$

Q1 Hint Have a go at solving these equations using a calculator too.

Q2 a) Multiply out $(x - 2 + \sqrt{5})(x - 2 - \sqrt{5})$.
b) Solve the equation $x^2 - 4x - 1 = 0$ using the quadratic formula.
c) How does your answer to b) relate to the expression given in a)?

Q3 The roots of the equation $x^2 + 8x + 13 = 0$ can be written in the form $x = A \pm \sqrt{B}$ where A and B are integers. Find A and B.

Q4 Solve the following equations, giving your answers in surd form where necessary.

a) $x^2 + x + \frac{1}{4} = 0$
b) $x^2 - \frac{7}{4}x + \frac{2}{3} = 0$
c) $25x^2 - 30x + 7 = 0$
d) $60x - 5 = -100x^2 - 3$
e) $2x(x - 4) = 7 - 3x$
f) $(3x - 5)(x + 2) = 3x - 2$

Completing the square (Ex. 4.3)

Q2 Rewrite the following expressions in the form $p(x + q)^2 + r$:

a) $x^2 + 6x + 8$
b) $x^2 + 8x - 10$
c) $x^2 - 3x - 10$
d) $x^2 - 20x + 15$
e) $x^2 - 2mx + n$
f) $x^2 + 6tx + s$
g) $3x^2 - 12x + 7$
h) $2x^2 - 4x - 3$
i) $6x^2 + 30x - 20$
j) $-x^2 - 9x + 9$
k) $4x^2 - 22x + 5$
l) $-3x^2 + 9x + 1$

Q3 Solve the following equations by completing the square:

a) $x^2 - 6x - 16 = 0$
b) $p^2 - 10p = 200$
d) $x^2 + 4x - 8 = 0$
e) $4x^2 + 24x - 13 = 0$

Q4 State the coordinates of the turning point on each of the following graphs. In each case, state whether it is a minimum or maximum.

a) $y = x^2 + 6x + 11$
b) $y = -x^2 - 9x + 9$
c) $y = 4x^2 - 22x + 5$

Sketching quadratic graphs (Ex. 4.7)

Q6 Sketch the following graphs, showing any intersections with the axes:

a) $y = x^2 - 2x + 1$
b) $y = x^2 + x - 1$
c) $y = x^2 - 8x + 18$
d) $y = -x^2 + 3$
e) $y = 2x^2 + 5x + 2$
f) $y = 2x^2 - 5x - 1$

Inequalities (Review ex. 5)

Q1 Solve:

a) $7x - 4 > 2x - 42$
b) $12y - 3 \leq 4y + 4$
c) $9y - 4 \geq 17y + 2$
d) $x + 6 < 5x - 4$
e) $4x - 2 > x - 14$
f) $7 - x \leq 4 - 2x$
g) $11x - 4 < 4 - 11x$
h) $1 + 10y \geq 7y - 12$
i) $8y - 6 \leq 6 - 8y$

Q2 Solve the inequality $3(2x - 5) + 2(4 - x) \geq x + 7$. *[3 marks]*

Q3 Find the set of values for x that satisfy the following inequalities:

a) $3x^2 - 5x - 2 \leq 0$
b) $x^2 + 2x + 7 > 4x + 9$
c) $3x^2 + 7x + 4 \geq 2(x^2 + x - 1)$
d) $x^2 + 3x - 1 \geq x + 2$
e) $2x^2 > x + 1$
f) $3x^2 - 12 < x^2 - 2x$
g) $3x^2 + 6x \leq 2x^2 + 3$
h) $(x + 2)(x - 3) \geq 8 - 3x^2$

Q6 Draw and shade the region which satisfies each of the following sets of inequalities.

a) $8 \leq y - x, \quad y < 12 - x, \quad 9x + 2y < -4$
b) $x + 3y > 15, \quad 3x + y < 12, \quad 4y \leq x + 36$

Simultaneous Equations

Q13 Solve these sets of simultaneous equations:

a) $3x - 4y = 7$ and $-2x + 7y = -22$
b) $2x - 3y = \frac{11}{12}$ and $x + y = -\frac{7}{12}$
c) $2x + 3y = 8$ and $6y = 5 - 4x$
d) $11y = 9x + 4$ and $3x - 2y = 7$
e) $\frac{1}{2}x + \frac{1}{3}y = 50$ and $x + 4y = 25$
f) $x + 4y = \frac{1}{4}$ and $y + 2x = \frac{1}{5}$

Q14 Find where the following lines meet:

a) $y = 3x - 4$ and $y = 7x - 5$
b) $y = 13 - 2x$ and $7x - y - 23 = 0$
c) $2x - 3y + 4 = 0$ and $x - 2y + 1 = 0$
d) $5x - 7y = 22$ and $3y - 4x - 13 = 0$

Simultaneous Equations – one linear, one quadratic (ex 5.5)

Q2 Solve the following simultaneous equations:

a) $y = 2x + 5$
$y = x^2 - x + 1$

b) $y = 2x^2 - 3$
$y = 3x + 2$

c) $2x^2 - xy = 6$
$y - 3x + 7 = 0$

d) $xy = 6$
$2y - x + 4 = 0$

f) $y = 2x^2 - 3x + 5$
$5x - y = 3$

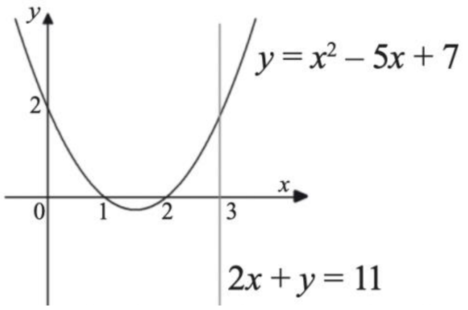
g) $2x^2 + 3y^2 + 18x = 347$
$4x + y = 7$

i) $x^2 + 4x = 4y + 40$
$12y + 5x + 30 = 0$

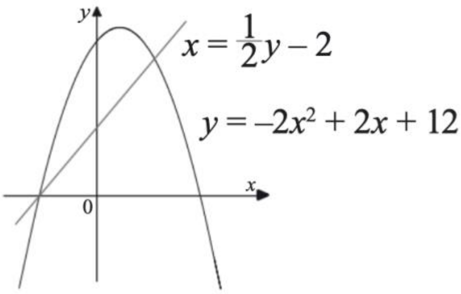
Q2 Hint For these sets of equations you'll need to do some rearranging.

Q4 Find the points of intersection between the lines and curves on the graphs below.

a)



b)



Straight Lines (rev ex 6)

Q1 Find the equations of the straight lines that pass through the following pairs of points. Write each of them in the forms (i) $y = mx + c$, (ii) $ax + by + c = 0$ where a, b and c are integers.

a) $(2, -1), (-4, -19)$ b) $\left(0, -\frac{1}{3}\right), \left(5, \frac{2}{3}\right)$ c) $(8,7), (-7, -2)$

Q2 Points A and B have coordinates $(-2, 4)$ and $(4, -10)$ respectively.

a) Find the exact length AB. *[2 marks]*

b) Find the gradient of the line segment AB. *[2 marks]*

c) Find the equation of the line that passes through points A and B, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found. *[3 marks]*

Q3 A triangle has vertices A, B and C. The coordinates of A and B are $(2, -1)$ and $(10, -1)$ respectively and C is a point on the line, l, with equation $y - 2x + 5 = 0$.

a) Show that A also lies on l. *[1 mark]*

b) Given that the x-coordinate of C is k, find the y-coordinate of C in terms of k. *[1 mark]*

The area of the triangle is 32 units^2.

c) Find the equation of the line through B and C in the form $y = mx + c$. *[3 marks]*

Q4 Determine whether the following lines are parallel to the line with equation $4x - 6y = 7$. You must give a suitable justification in each case.

a) $8x + 12y = 15$ *[2 marks]*

b) $3y - 2x = 7$ *[2 marks]*

c) $y = \dfrac{4x + 3}{6}$ *[2 marks]*

Q6 a) The line l_1 has equation $y = \frac{3}{2}x - \frac{2}{3}$. Find the equation of the line parallel to l_1, passing through the point with coordinates $(4, 2)$.

b) The line l_2 passes through the point $(6, 1)$ and is perpendicular to $2x - y - 7 = 0$. Find the equation of the line l_2.

Q7 The coordinates of points R and S are $(1, 9)$ and $(10, 3)$ respectively. Find the equation of the line perpendicular to RS, passing through the point $(1, 9)$.

Graph Transformations (rev ex 6)

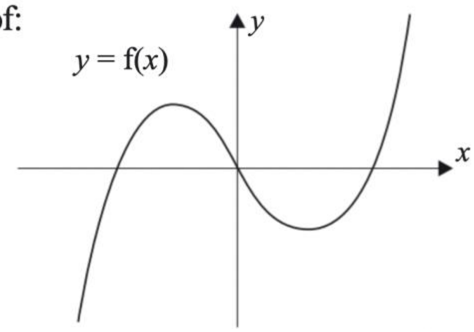
Q16 Given that $a > 1$, use the graph of $f(x)$ to sketch the graph of:

a) $y = f(ax)$ b) $y = f\left(\frac{1}{a}x\right)$

c) $y = af(x)$ d) $y = \frac{1}{a}f(x)$

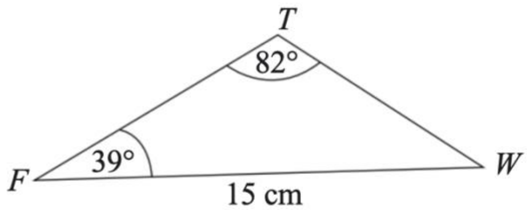
e) $y = f(x + a)$ f) $y = f(x - a)$

g) $y = f(x) + a$ h) $y = f(x) - a$



Trigonometry ex 8.1

Q6 Use the sine rule to find the length TW.

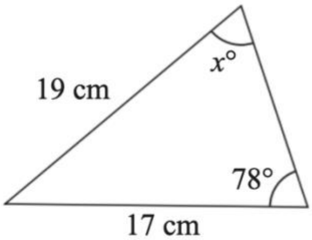


Q7 In triangle PQR: $PR = 48$ m, angle $P = 38°$ and angle $R = 43°$. Find the length PQ.

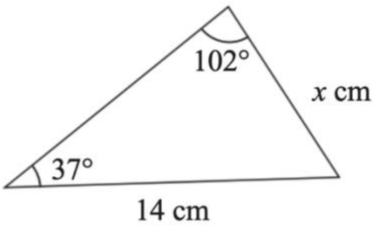
Q8 In triangle ABC: $AB = 17$ cm, $AC = 14$ cm and angle $C = 67°$. Find the angle A.

Q9 For the following triangles, find the missing value x.

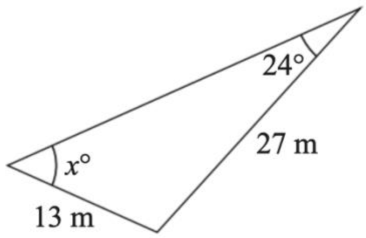
a)



b)



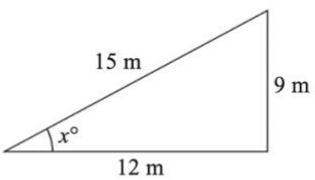
c)



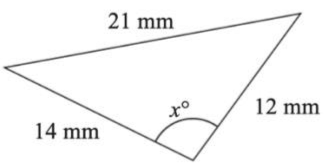
Ex 8.4

Q3 For the following triangles, find the missing value x.

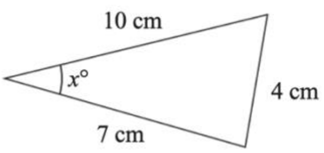
a)



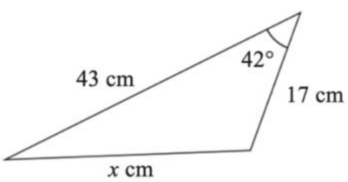
b)



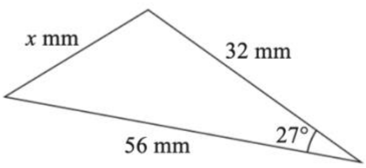
c)



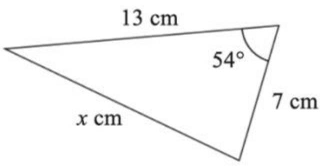
d)



e)



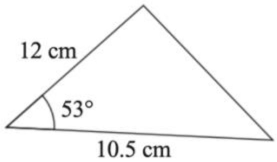
f)



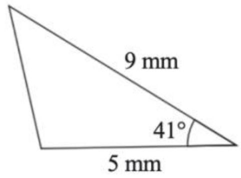
Ex 8.5

Q1 Find the area of the following triangles.

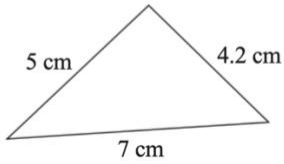
a)



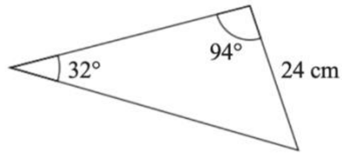
b)



c)



d)



Q2 In triangle PQR: $PQ = 4$ cm, $QR = 7$ cm and angle $Q = 49°$. Find the area of triangle PQR.

Trig Graphs Ex 8.7

Q1 On the same set of axes, sketch the graphs of
$y = \cos x$ and $y = \cos x + 3$ in the interval $-360° \leq x \leq 360°$.

Q2 On the same set of axes, sketch the graphs of
$y = \cos x$ and $y = \cos(x + 90°)$ in the interval $-180° \leq x \leq 180°$.

Q3 On the same set of axes, sketch the graphs of
$y = \tan x$ and $y = \tan(x - 45°)$ in the interval $-180° \leq x \leq 360°$.

Q4 On the same set of axes, sketch the graphs of
$y = \sin x$ and $y = \frac{1}{3}\sin x$ in the interval $-180° \leq x \leq 180°$.

Q5 On the same set of axes, sketch the graphs of $y = \sin x$ and $y = \sin 3x$ in the interval $0° \leq x \leq 360°$.

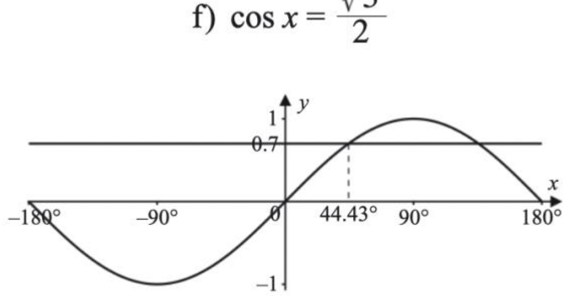
Using trig graphs to solve equations Ex 8.8

Q3 By sketching a graph, find all the solutions to the equations below in the interval $0° \leq x \leq 360°$.

a) $\cos x = \frac{1}{\sqrt{2}}$ b) $\tan x = \sqrt{3}$ c) $\sin x = \frac{1}{2}$

d) $\tan x = \frac{1}{\sqrt{3}}$ e) $\tan x = 1$ f) $\cos x = \frac{\sqrt{3}}{2}$

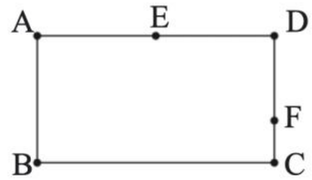
Q4 One solution of $\sin x = 0.7$ is 44.43° (2 d.p.).
Use the graph to find all the solutions in the interval $-180° \leq x \leq 180°$.



Vectors Ex 12.1

Q7 In the rectangle ABCD, E is the midpoint of AD and F divides DC in the ratio 2 : 1. If $\overrightarrow{AB} = \mathbf{b}$ and $\overrightarrow{AD} = \mathbf{d}$, find the following vectors in terms of $\mathbf{b}$ and $\mathbf{d}$.

a) $\overrightarrow{DF}$ b) $\overrightarrow{BE}$ c) $\overrightarrow{EF}$

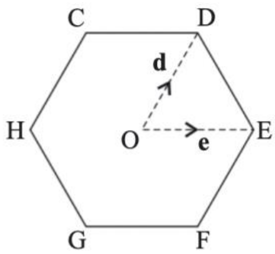


Q8 CDEFGH is a regular hexagon whose centre is O. If $\overrightarrow{OE} = \mathbf{e}$ and $\overrightarrow{OD} = \mathbf{d}$, express in terms of $\mathbf{e}$ and $\mathbf{d}$:

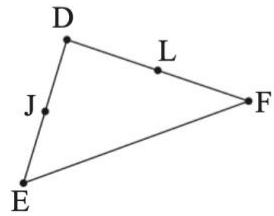
a) $\overrightarrow{HE}$ b) $\overrightarrow{DG}$

c) $\overrightarrow{ED}$ d) $\overrightarrow{CE}$

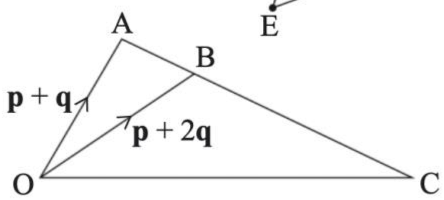
e) $\overrightarrow{DF}$ f) $\overrightarrow{EG}$



Q9 In triangle DEF, J and L are midpoints of ED and FD respectively. Given that $\overrightarrow{EF} = \mathbf{f}$ and $\overrightarrow{ED} = \mathbf{d}$, prove that $\overrightarrow{JL} = \frac{1}{2}\mathbf{f}$.



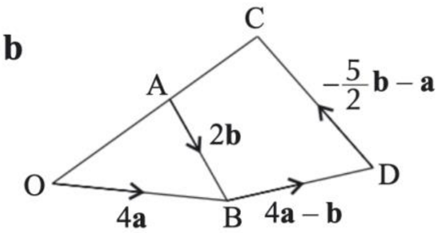
Q10 In the diagram on the right, BC = 3AB. $\overrightarrow{AB}$ and $\overrightarrow{BC}$ lie along the same straight line. Find the following vectors in terms of $\mathbf{p}$ and $\mathbf{q}$:



a) $\overrightarrow{AB}$ *[1 mark]*

b) $\overrightarrow{OC}$ *[2 marks]*

Q20 In the diagram on the right, $\overrightarrow{OB} = 4\mathbf{a}$, $\overrightarrow{AB} = 2\mathbf{b}$, $\overrightarrow{BD} = 4\mathbf{a} - \mathbf{b}$ and $\overrightarrow{DC} = -\frac{5}{2}\mathbf{b} - \mathbf{a}$. Show that OAC is a straight line.

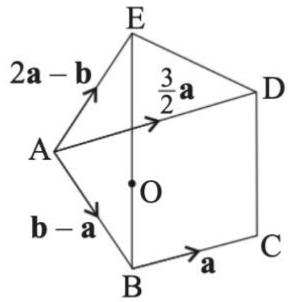


Q21 $\overrightarrow{AB} = \mathbf{q} - \frac{1}{2}\mathbf{p}$, $\overrightarrow{AC} = \mathbf{p}$, $\overrightarrow{AD} = -5\mathbf{p} + 4\mathbf{q}$.
Show that B, C and D are collinear.

Q24 In the diagram on the right, $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$, $\overrightarrow{BC} = \mathbf{a}$, $\overrightarrow{AD} = \frac{3}{2}\mathbf{a}$ and $\overrightarrow{AE} = 2\mathbf{a} - \mathbf{b}$.

a) Show that BE is parallel to CD.

b) The point O divides $\overrightarrow{BE}$ in the ratio 1 : 2.
Show that AOC is a straight line.



Worked Solutions

Chapter 3 Review Exercise

Q1 **a)** $(a+b)(a-b) = a^2 - ab + ba - b^2 = a^2 - b^2$

b) $(p+q)(p+q) = p^2 + pq + qp + q^2 = p^2 + 2pq + q^2$

c) $35xy + 25y(5y+7x) - 100y^2$
$= 35xy + 125y^2 + 175xy - 100y^2 = 25y^2 + 210xy$

d) $(x+3y+2)(3x+y+7)$
$= x(3x+y+7) + 3y(3x+y+7) + 2(3x+y+7)$
$= 3x^2 + xy + 7x + 9xy + 3y^2 + 21y + 6x + 2y + 14$
$= 3x^2+10xy+3y^2+13x+23y+14$

e) $(c+2d)(c-d)(2d-3c)$
$= (c^2 - cd + 2dc - 2d^2)(2d-3c)$
$= c^2(2d-3c) - cd(2d-3c) + 2dc(2d-3c) - 2d^2(2d-3c)$
$= 2c^2d - 3c^3 - 2cd^2 + 3c^2d + 4cd^2 - 6c^2d - 4d^3 + 6cd^2$
$= -c^2d - 3c^3 + 8cd^2 - 4d^3$

f) $[(s-2t)(s+2t)]^2 = [s^2 + 2st - 2ts - 4t^2]^2$
$= [s^2 - 4t^2]^2 = (s^2 - 4t^2)(s^2 - 4t^2)$
$= s^4 - 4s^2t^2 - 4t^2s^2 + 16t^4 = s^4 - 8s^2t^2 + 16t^4$

Q2 **a)** $(2x^2 + 7x - 8)(6x^2 - 5x - 10)$
$= 12x^4 - 10x^3 - 20x^2 + 42x^3 - 35x^2 - 70x - 48x^2 + 40x + 80$
$= 12x^4 + 32x^3 - 103x^2 - 30x + 80$

b) $3p(q-5r) + 4q(p+2r) - 7r(2p-3q)$
$= 3pq - 15pr + 4pq + 8qr - 14pr + 21qr$
$= 7pq - 29pr + 29qr$

Q3 The side lengths of the box are x cm, $(x-10)$ cm and $(x-8)$ cm.
So the volume of one box is $x(x-10)(x-8)$ cm^3.
$x(x-10)(x-8) = x(x^2 - 8x - 10x + 80)$
$= x^3 - 18x^2 + 80x$ cm^3
So the volume of 5 boxes is:
$5(x^3 - 18x^2 + 80x) = 5x^3 - 90x^2 + 400x$ cm^3

Q4 $p(x) - (2-3x)q(x) = 3x^3 - 7x + 5 - (2-3x)(x^2 - 3x + 1)$
$= 3x^3 - 7x + 5 - (2x^2 - 6x + 2 - 3x^3 + 9x^2 - 3x)$
$= 6x^3 - 11x^2 + 2x + 3 \Rightarrow a = 6, b = -11, c = 2$ and $d = 3$
[3 marks available — 1 mark for correctly expanding brackets, 1 mark for correctly simplifying, 1 mark for all four correct values of a, b, c and d]

Ex 3.3 Algebraic fractions

Q2 **a)** The common denominator is $(y-1)(y-2)$:

$$\frac{5}{y-1} + \frac{3}{y-2} = \frac{5(y-2)}{(y-1)(y-2)} + \frac{3(y-1)}{(y-1)(y-2)} = \frac{5(y-2)+3(y-1)}{(y-1)(y-2)} = \frac{5y-10+3y-3}{(y-1)(y-2)} = \frac{8y-13}{(y-1)(y-2)}$$

b) The common denominator is $(r-5)(r+3)$:

$$\frac{7}{r-5} - \frac{4}{r+3} = \frac{7(r+3)}{(r-5)(r+3)} - \frac{4(r-5)}{(r-5)(r+3)} = \frac{7(r+3)-4(r-5)}{(r-5)(r+3)} = \frac{7r+21-4r+20}{(r-5)(r+3)} = \frac{3r+41}{(r-5)(r+3)}$$

c) The common denominator is $p(p-3)$:

$$\frac{8}{p} - \frac{1}{p-3} = \frac{8(p-3)}{p(p-3)} - \frac{p}{p(p-3)} = \frac{8p-24-p}{p(p-3)} = \frac{7p-24}{p(p-3)}$$

d) The common denominator is $2(w-2)(w-7)$:

$$\frac{w}{2(w-2)} + \frac{3w}{w-7}$$
$$= \frac{w(w-7)}{2(w-2)(w-7)} + \frac{3w \times 2(w-2)}{2(w-2)(w-7)}$$
$$= \frac{w^2-7w}{2(w-2)(w-7)} + \frac{6w(w-2)}{2(w-2)(w-7)}$$
$$= \frac{w^2-7w+6w(w-2)}{2(w-2)(w-7)}$$
$$= \frac{w^2-7w+6w^2-12w}{2(w-2)(w-7)}$$
$$= \frac{7w^2-19w}{2(w-2)(w-7)} = \frac{w(7w-19)}{2(w-2)(w-7)}$$

e) The common denominator is $(z+2)(z+4)$:

$$\frac{z+1}{z+2} - \frac{z+3}{z+4} = \frac{(z+1)(z+4)}{(z+2)(z+4)} - \frac{(z+2)(z+3)}{(z+2)(z+4)}$$
$$= \frac{(z+1)(z+4)-(z+2)(z+3)}{(z+2)(z+4)}$$
$$= \frac{(z^2+5z+4)-(z^2+5z+6)}{(z+2)(z+4)}$$
$$= \frac{-2}{(z+2)(z+4)}$$

f) The common denominator is $(q+1)(q-2)$:

$$\frac{1}{q+1}+\frac{3}{q-2}=\frac{(q-2)}{(q+1)(q-2)}+\frac{3(q+1)}{(q+1)(q-2)}$$
$$=\frac{(q-2)+3(q+1)}{(q+1)(q-2)}$$
$$=\frac{q-2+3q+3}{(q+1)(q-2)}$$
$$=\frac{4q+1}{(q+1)(q-2)}$$

g) The common denominator is $(x+z)(x-z)$:

$$\frac{x}{x+z}+\frac{2z}{x-z}=\frac{x(x-z)}{(x+z)(x-z)}+\frac{2z(x+z)}{(x+z)(x-z)}$$
$$=\frac{x^2-xz+2xz+2z^2}{(x+z)(x-z)}=\frac{x^2+xz+2z^2}{(x+z)(x-z)}$$

h) The common denominator is $(2x+3)(3-x)$:

$$\frac{y}{2x+3}-\frac{2y}{3-x}=\frac{y(3-x)}{(2x+3)(3-x)}-\frac{2y(2x+3)}{(2x+3)(3-x)}$$
$$=\frac{3y-xy-4xy-6y}{(2x+3)(3-x)}$$
$$=\frac{-3y-5xy}{(2x+3)(3-x)}=\frac{3y+5xy}{(2x+3)(x-3)}$$

In the final step, top and bottom have been multiplied by −1 to make the answer a bit neater.

i) The common denominator is $r(r-4)(r+1)$:

$$\frac{5}{r-4}+\frac{3}{r}-\frac{r}{r+1}$$
$$=\frac{5r(r+1)}{r(r-4)(r+1)}+\frac{3(r-4)(r+1)}{r(r-4)(r+1)}-\frac{r^2(r-4)}{r(r-4)(r+1)}$$
$$=\frac{5r^2+5r+3(r^2+r-4r-4)-(r^3-4r^2)}{r(r-4)(r+1)}$$
$$=\frac{5r^2+5r+3r^2-9r-12-r^3+4r^2}{r(r-4)(r+1)}$$
$$=\frac{-r^3+12r^2-4r-12}{r(r-4)(r+1)}$$

Q15 Call the unknown side of the flower bed z.

Then $z=\frac{x^2}{x-3}$ and $y+z=\frac{3x^2}{x+6}$.

So $y=\frac{3x^2}{x+6}-z=\frac{3x^2}{x+6}-\frac{x^2}{x-3}=\frac{3x^2(x-3)-x^2(x+6)}{(x+6)(x-3)}$

$$=\frac{3x^3-9x^2-x^3-6x^2}{(x+6)(x-3)}=\frac{2x^3-15x^2}{(x+6)(x-3)}=\frac{x^2(2x-15)}{(x+6)(x-3)}$$

Indices

Q16 **a)** $x^3 \cdot x^5 = x^{3+5} = x^8$ **b)** $a^7 \cdot a^8 = a^{7+8} = a^{15}$

c) $\frac{x^8}{x^2} = x^{8-2} = x^6$ **d)** $(a^2)^4 = a^{2\times4} = a^8$

e) $(xy^2)\cdot(x^3yz) = x^{1+3}y^{2+1}z = x^4y^3z$

f) $\frac{a^2b^4c^6}{a^3b^2c} = a^{2-3}b^{4-2}c^{6-1} = a^{-1}b^2c^5 = \frac{b^2c^5}{a}$

Q17 **a)** $g^2 \times g^{-5} = g^{2-5} = g^{-3}$

b) $p^4r^2 \div p^5r^{-6} = p^{4-5}r^{2-(-6)} = p^{-1}r^8$

c) $\left(k^{\frac{1}{3}}\right)^6 = k^{\frac{1}{3}\times 6} = k^2$

d) $(mn^8 \times m^4n^{-11})^{-2} = (m^{1+4}n^{8-11})^{-2} = (m^5n^{-3})^{-2}$
$= m^{5\times(-2)}n^{(-3)\times(-2)} = m^{-10}n^6$

e) $s^4t^3\left(\frac{1}{s^2t^5}\right)^{-3} = s^4t^3(s^2t^5)^3 = s^4t^3(s^2)^3(t^5)^3 = s^4t^3s^{2\times3}t^{5\times3}$
$= s^4t^3s^6t^{15} = s^{4+6}t^{3+15} = s^{10}t^{18}$

f) $\frac{a^2}{b^2c}\times\frac{b^6}{a^4c^{-2}}\div\frac{c^2}{a^3b} = \frac{a^2}{b^2c}\times\frac{b^6}{a^4c^{-2}}\times\frac{a^3b}{c^2}$
$= \frac{a^{2+3}b^{6+1}}{a^4b^2c^{1-2+2}} = \frac{a^5b^7}{a^4b^2c} = a^{5-4}b^{7-2}c^{-1} = ab^5c^{-1}$

Q18 $\frac{(3a^3b^2)^2\times(2a^2b)^2}{(8a^6b^{-3})^{\frac{1}{3}}} = \frac{9a^6b^4\times4a^4b^2}{8^{\frac{1}{3}}a^2b^{-1}} = \frac{36a^{10}b^6}{2a^2b^{-1}} = 18a^8b^7$

Q19 **a)** Write the RHS as 9 to the power 'something': $3=\sqrt{9}=9^{\frac{1}{2}}$

So $x=\frac{1}{2}$ *[1 mark]*

b) $9^{3x} = (3^2)^{3x} = 3^{6x}$

$81 = 9^2 = 3^4$, so $81^{2x-1} = (3^4)^{2x-1} = 3^{8x-4}$

[1 mark for expressing both terms on the left-hand side of the equation as powers of 3]

So $9^{3x}\cdot 81^{2x-1} = 3^{6x}\times 3^{8x-4} = 3^{6x+8x-4} = 3^{14x-4}$

So $9^{3x}\cdot 81^{2x-1} = 27 \Rightarrow 3^{14x-4} = 3^3$

[1 mark for writing both sides of the equation as a single power of 3]

So $14x-4=3 \Rightarrow 14x=7 \Rightarrow x=\frac{1}{2}$

[1 mark for the correct value of x]

Q20 **a)** $16^{\frac{1}{2}}=\sqrt{16}=4$ **b)** $8^{\frac{1}{3}}=\sqrt[3]{8}=2$

c) $81^{\frac{3}{4}}=\left(81^{\frac{1}{4}}\right)^3=3^3=27$ **d)** $x^0=1$

e) $49^{-\frac{1}{2}}=\frac{1}{\sqrt{49}}=\frac{1}{7}$

f) $\frac{1}{27^{-\frac{2}{3}}}=27^{\frac{2}{3}}=\left(27^{\frac{1}{3}}\right)^2=3^2=9$

Surds

Q24 **a)** $\sqrt{28} = \sqrt{4 \times 7} = \sqrt{4}\sqrt{7} = 2\sqrt{7}$

b) $\sqrt{\frac{5}{36}} = \frac{\sqrt{5}}{\sqrt{36}} = \frac{\sqrt{5}}{6}$

c) $\sqrt{18} = \sqrt{9 \times 2} = \sqrt{9}\sqrt{2} = 3\sqrt{2}$

d) $\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$

Q25 **a)** $\sqrt{3} - \sqrt{12} = \sqrt{3} - \sqrt{4 \times 3} = \sqrt{3} - 2\sqrt{3} = -\sqrt{3}$

b) $3\sqrt{5} + \sqrt{45} = 3\sqrt{5} + \sqrt{9 \times 5} = 3\sqrt{5} + 3\sqrt{5} = 6\sqrt{5}$

c) $\sqrt{7} + \sqrt{448} = \sqrt{7} + \sqrt{64 \times 7} = \sqrt{7} + 8\sqrt{7} = 9\sqrt{7}$

d) $\sqrt{52} + \sqrt{117} = \sqrt{4 \times 13} + \sqrt{9 \times 13}$
$= 2\sqrt{13} + 3\sqrt{13} = 5\sqrt{13}$

e) $4\sqrt{150} + \sqrt{54} - \sqrt{5}\sqrt{120}$
$= 4\sqrt{25 \times 6} + \sqrt{9 \times 6} - \sqrt{5}\sqrt{20 \times 6}$
$= 4 \times 5\sqrt{6} + 3\sqrt{6} - \sqrt{5}\sqrt{4 \times 5}\sqrt{6}$
$= 20\sqrt{6} + 3\sqrt{6} - 2\sqrt{5}\sqrt{5}\sqrt{6}$
$= 20\sqrt{6} + 3\sqrt{6} - 10\sqrt{6} = 13\sqrt{6}$

Q26 The larger square has side length $\sqrt{1920}$ cm and the smaller square has side length $\sqrt{1080}$ cm.
a is the difference between the side lengths of the two squares, so:
$a = \sqrt{1920} - \sqrt{1080} = \sqrt{64 \times 30} - \sqrt{36 \times 30}$
$= 8\sqrt{30} - 6\sqrt{30} = 2\sqrt{30}$

Q30 $(3\sqrt{5} - 5\sqrt{3})^2 = (3\sqrt{5} - 5\sqrt{3})(3\sqrt{5} - 5\sqrt{3})$
$= 9\sqrt{5}\sqrt{5} - 15\sqrt{5}\sqrt{3} - 15\sqrt{3}\sqrt{5} + 25\sqrt{3}\sqrt{3}$
$= 45 - 15\sqrt{15} - 15\sqrt{15} + 75 = 120 - 30\sqrt{15} = 30(4 - \sqrt{15})$
[3 marks available — 3 marks for the correct answer, otherwise 2 marks for any two correctly simplified terms (45, $-15\sqrt{15}$ or 75), or 1 mark for attempting to expand the brackets and getting one term correct]

Q31 **a)** $\frac{8}{\sqrt{2}} = \frac{8}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$

b) $\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{(\sqrt{2})^2} = \frac{1}{\sqrt{2}}$

Q32 $\frac{2}{3+\sqrt{7}} = \frac{2(3-\sqrt{7})}{(3+\sqrt{7})(3-\sqrt{7})}$
$= \frac{6-2\sqrt{7}}{9-7} = \frac{6-2\sqrt{7}}{2} = 3 - \sqrt{7}$

Q33 **a)** $(2\sqrt{2} + \sqrt{3})^2 = 4 \times 2 + 4\sqrt{2}\sqrt{3} + 3 = 11 + 4\sqrt{6}$
[2 marks available — 1 mark for correctly expanding the brackets, 1 mark for the correct simplified answer]

b) Using the result from a), $\frac{1}{(2\sqrt{2} + \sqrt{3})^2} = \frac{1}{11 + 4\sqrt{6}}$
$= \frac{1}{11+4\sqrt{6}} \times \frac{11-4\sqrt{6}}{11-4\sqrt{6}} = \frac{11-4\sqrt{6}}{121-96} = \frac{11-4\sqrt{6}}{25}$
[2 marks available — 2 marks for the correct answer, otherwise 1 mark for using the correct method]

Q34 **a)** $\frac{11+\sqrt{13}}{5-\sqrt{13}} = \frac{(11+\sqrt{13})(5+\sqrt{13})}{(5-\sqrt{13})(5+\sqrt{13})}$
$= \frac{55 + 11\sqrt{13} + 5\sqrt{13} + 13}{25 - 13}$
$= \frac{68 + 16\sqrt{13}}{12} = \frac{17}{3} + \frac{4}{3}\sqrt{13}$

b) $\frac{2\sqrt{7}+9}{3-\sqrt{7}} = \frac{(2\sqrt{7}+9)(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})}$
$= \frac{6\sqrt{7} + 14 + 27 + 9\sqrt{7}}{9 - 7}$
$= \frac{15\sqrt{7} + 41}{2} = \frac{41}{2} + \frac{15}{2}\sqrt{7}$

c) $\frac{3\sqrt{5}+\sqrt{15}}{\sqrt{60}-\sqrt{20}} = \frac{(3\sqrt{5}+\sqrt{15})(\sqrt{60}+\sqrt{20})}{(\sqrt{60}-\sqrt{20})(\sqrt{60}+\sqrt{20})}$
$= \frac{3\sqrt{5}\sqrt{60} + 3\sqrt{5}\sqrt{20} + \sqrt{15}\sqrt{60} + \sqrt{15}\sqrt{20}}{60 - 20}$
$= \frac{3\sqrt{5}\sqrt{5}\sqrt{3}\sqrt{4} + 3\sqrt{5}\sqrt{5}\sqrt{4}}{40}$
$+ \frac{\sqrt{3}\sqrt{5}\sqrt{5}\sqrt{3}\sqrt{4} + \sqrt{3}\sqrt{5}\sqrt{5}\sqrt{4}}{40}$
$= \frac{30\sqrt{3} + 30 + 30 + 10\sqrt{3}}{40}$
$= \frac{40\sqrt{3} + 60}{40} = \frac{3}{2} + \sqrt{3}$

Q35 $(\sqrt{32} + \sqrt{128})^2 = (4\sqrt{2} + 8\sqrt{2})^2 = (12\sqrt{2})^2 = 144 \times 2 = 288$
$\Rightarrow \frac{(\sqrt{32}+\sqrt{128})^2}{\sqrt{5}+\sqrt{3}} = \frac{288}{\sqrt{5}+\sqrt{3}} = \frac{288}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ $= \frac{288(\sqrt{5}-\sqrt{3})}{5-3} = 144(\sqrt{5}-\sqrt{3})$ or $144\sqrt{5} - 144\sqrt{3}$

Exercise 4.1 — Factorising a Quadratic

Q1 **a)** $x^2 - 6x + 5 = (x - 5)(x - 1)$

b) $x^2 - 3x - 18 = (x - 6)(x + 3)$

c) $x^2 + 22x + 121 = (x + 11)(x + 11) = (x + 11)^2$

d) $x^2 - 12x = x(x - 12)$

Note that if every term contains an x, you can just take a factor of x out of the bracket.

e) $y^2 - 13y + 42 = (y - 6)(y - 7)$

f) $x^2 + 51x + 144 = (x + 48)(x + 3)$

g) $x^2 - 121 = (x + 11)(x - 11)$

If there is no 'b' term, see if the expression is a 'difference of two squares' (chances are it will be).

h) $x^2 - 35x + 66 = (x - 2)(x - 33)$

Q2 **a)** $x^2 - 3x - 10 = 0 \Rightarrow (x - 5)(x + 2) = 0$
$\Rightarrow x - 5 = 0$ or $x + 2 = 0 \Rightarrow x = 5$ or $x = -2$

b) $2x^2 + 2x - 40 = 0 \Rightarrow 2(x^2 + x - 20) = 0$

This is an example of a question where you can simplify the equation before factorising. You can divide through by 2.

$x^2 + x - 20 = 0 \Rightarrow (x + 5)(x - 4) = 0$
$\Rightarrow x + 5 = 0$ or $x - 4 = 0 \Rightarrow x = -5$ or $x = 4$

c) $p^2 + 21p + 38 = 0 \Rightarrow (p + 19)(p + 2) = 0$
$\Rightarrow p + 19 = 0$ or $p + 2 = 0 \Rightarrow p = -19$ or $p = -2$

d) $x^2 - 15x + 54 = 0 \Rightarrow (x - 9)(x - 6) = 0$
$\Rightarrow x - 9 = 0$ or $x - 6 = 0 \Rightarrow x = 9$ or $x = 6$

e) $x^2 + 18x = -65 \Rightarrow x^2 + 18x + 65 = 0$
$(x + 5)(x + 13) = 0 \Rightarrow x + 5 = 0$ or $x + 13 = 0$
$\Rightarrow x = -5$ or $x = -13$

f) $x^2 - x = 42 \Rightarrow x^2 - x - 42 = 0 \Rightarrow (x - 7)(x + 6) = 0$
$\Rightarrow x - 7 = 0$ or $x + 6 = 0 \Rightarrow x = 7$ or $x = -6$

g) $x^2 + 1100x + 100\,000 = 0 \Rightarrow (x + 100)(x + 1000) = 0$
$\Rightarrow x + 100 = 0$ or $x + 1000 = 0 \Rightarrow x = -100$ or $x = -1000$

h) $3x^2 - 3x - 6 = 0 \Rightarrow 3(x^2 - x - 2) = 0$
$\Rightarrow x^2 - x - 2 = 0 \Rightarrow (x - 2)(x + 1) = 0$
$\Rightarrow x - 2 = 0$ or $x + 1 = 0 \Rightarrow x = 2$ or $x = -1$

Q3 **a)** $4x^2 - 4x - 3 = (2x + 1)(2x - 3)$

b) $2x^2 + 23x + 11 = (2x + 1)(x + 11)$

c) $7x^2 - 19x - 6 = (7x + 2)(x - 3)$

d) $-x^2 - 5x + 36 = -(x^2 + 5x - 36) = -(x - 4)(x + 9)$

e) $6x^2 - 7x - 3 = (3x + 1)(2x - 3)$

f) $2x^2 - 2 = 2(x^2 - 1) = 2(x + 1)(x - 1)$

g) $3x^2 - 3 = 3(x^2 - 1) = 3(x + 1)(x - 1)$

h) $-x^2 + 9x - 14 = -(x^2 - 9x + 14) = -(x - 7)(x - 2)$

Quadratic Formula Ex 4.2

1a) $x = 2 \pm \sqrt{2}$

1b) $x = 1 \pm 3\sqrt{5}$

1c) $x = -\frac{3}{2} \pm \frac{1}{2}\sqrt{57}$

1d) $x = 7 \pm \sqrt{7}$

1e) $x = -\frac{1}{2} \pm \frac{1}{2}\sqrt{2}$

1f) $x = 1$ or 3

1g) $x = \frac{1}{2}$ or $\frac{1}{3}$

1h) $x = \sqrt{11}$

Q2 **a)** $(x - 2 + \sqrt{5})(x - 2 - \sqrt{5})$
$= x(x - 2 - \sqrt{5}) - 2(x - 2 - \sqrt{5}) + \sqrt{5}(x - 2 - \sqrt{5})$
$= x^2 - 2x - \sqrt{5}x - 2x + 4 + 2\sqrt{5} + \sqrt{5}x - 2\sqrt{5} - 5$
$= x^2 - 4x - 1$

Use the method for multiplying out long brackets from Chapter 3.

b) $x^2 - 4x - 1 = 0 \Rightarrow a = 1, b = -4, c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times (-1)}}{2 \times 1} = \frac{4 \pm \sqrt{16 + 4}}{2} = \frac{4 \pm \sqrt{20}}{2} = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

c) The roots produced by the quadratic formula in part b) are the same as the numbers subtracted from x in the expression from a) — this is because it's just the factorised version of the same quadratic. If you put the factorised version equal to zero and solved the equation, you'd get the same roots.

Q3 $x^2 + 8x + 13 = 0 \Rightarrow a = 1, b = 8, c = 13$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8 \pm \sqrt{8^2 - 4 \times 1 \times 13}}{2 \times 1} = \frac{-8 \pm \sqrt{64 - 52}}{2} = \frac{-8 \pm \sqrt{12}}{2} = \frac{-8 \pm 2\sqrt{3}}{2} = -4 \pm \sqrt{3}$$

So A = – 4 and B = 3.

4a) $x = -\frac{1}{2}$

4b) $x = \frac{7}{8} \pm \frac{1}{24}\sqrt{57}$

4c) $x = \frac{3}{5} \pm \frac{1}{5}\sqrt{2}$

4d) $x = -\frac{3}{10} \pm \frac{1}{10}\sqrt{11}$

4e) $x = \frac{7}{2}$ or -1

4f) $x = 2$ or $-\frac{4}{3}$

Completing the Square Ex 4.3

Q2 **a)** $x^2 + 6x + 8 = (x+3)^2 - 9 + 8 = (x+3)^2 - 1$

b) $x^2 + 8x - 10 = (x+4)^2 - 16 - 10 = (x+4)^2 - 26$

c) $x^2 - 3x - 10 = \left(x - \frac{3}{2}\right)^2 - \frac{9}{4} - 10$
$= \left(x - \frac{3}{2}\right)^2 - \frac{9}{4} - \frac{40}{4} = \left(x - \frac{3}{2}\right)^2 - \frac{49}{4}$

d) $x^2 - 20x + 15 = (x-10)^2 - 100 + 15 = (x-10)^2 - 85$

e) $x^2 - 2mx + n = (x-m)^2 - m^2 + n = (x-m)^2 + (-m^2 + n)$

f) $x^2 + 6tx + s = (x+3t)^2 - 9t^2 + s = (x+3t)^2 + (-9t^2 + s)$

g) $3x^2 - 12x + 7 = 3(x-2)^2 - 12 + 7 = 3(x-2)^2 - 5$

h) $2x^2 - 4x - 3 = 2(x-1)^2 - 2 - 3 = 2(x-1)^2 - 5$

i) $6x^2 + 30x - 20 = 6\left(x + \frac{5}{2}\right)^2 - \frac{75}{2} - 20 = 6\left(x + \frac{5}{2}\right)^2 - \frac{115}{2}$

j) $-x^2 - 9x + 9 = -\left(x + \frac{9}{2}\right)^2 + \frac{81}{4} + 9 = -\left(x + \frac{9}{2}\right)^2 + \frac{117}{4}$

k) $4x^2 - 22x + 5 = 4\left(x - \frac{11}{4}\right)^2 - \frac{121}{4} + 5 = 4\left(x - \frac{11}{4}\right)^2 - \frac{101}{4}$

l) $-3x^2 + 9x + 1 = -3\left(x - \frac{3}{2}\right)^2 + \frac{27}{4} + 1 = -3\left(x - \frac{3}{2}\right)^2 + \frac{31}{4}$

Q3 **a)** First complete the square of the expression:
$x^2 - 6x - 16 = (x-3)^2 - 9 - 16 = (x-3)^2 - 25$
Now set the completed square equal to zero:
$(x-3)^2 - 25 = 0 \Rightarrow (x-3)^2 = 25$
$\Rightarrow x - 3 = \pm\sqrt{25} \Rightarrow x = 3 \pm \sqrt{25} = 3 \pm 5 \Rightarrow x = 8$ or -2

b) Write the equation in standard quadratic form:
$p^2 - 10p = 200 \Rightarrow p^2 - 10p - 200 = 0$
Then complete the square of the expression:
$p^2 - 10p - 200 = (p-5)^2 - 25 - 200 = (p-5)^2 - 225$
Now set the completed square equal to zero:
$(p-5)^2 - 225 = 0 \Rightarrow (p-5)^2 = 225 \Rightarrow p - 5 = \pm\sqrt{225}$
$\Rightarrow p = 5 \pm \sqrt{225} = 5 \pm 15 \Rightarrow p = 20$ or -10

c) First complete the square of the expression:
$x^2 + 2x + k = (x+1)^2 - 1 + k = (x+1)^2 + (k-1)$
Now set the completed square equal to zero:
$(x+1)^2 + (k-1) = 0 \Rightarrow (x+1)^2 = 1 - k$
$\Rightarrow x + 1 = \pm\sqrt{1-k} \Rightarrow x = -1 \pm \sqrt{1-k}$

d) First complete the square of the expression:
$x^2 + 4x - 8 = (x+2)^2 - 4 - 8 = (x+2)^2 - 12$
Now set the completed square equal to zero:
$(x+2)^2 - 12 = 0 \Rightarrow (x+2)^2 = 12$
$\Rightarrow x + 2 = \pm\sqrt{12} \Rightarrow x = \pm\sqrt{12} - 2$. So $x = -2 \pm 2\sqrt{3}$

e) First complete the square of the expression:
$4x^2 + 24x - 13 = 4(x+3)^2 - 36 - 13 = 4(x+3)^2 - 49$
Now set the completed square equal to zero:
$4(x+3)^2 - 49 = 0 \Rightarrow 4(x+3)^2 = 49 \Rightarrow x + 3 = \pm\sqrt{\frac{49}{4}}$
$\Rightarrow x = -3 \pm \sqrt{\frac{49}{4}} \Rightarrow x = -3 \pm \frac{7}{2}$, so $x = \frac{1}{2}$ or $-\frac{13}{2}$

f) Write the equation in standard quadratic form:
$9x^2 + 18x = 16 \Rightarrow 9x^2 + 18x - 16 = 0$
Then complete the square of the expression:
$9x^2 + 18x - 16 = 9(x+1)^2 - 9 - 16 = 9(x+1)^2 - 25$
Now set the completed square equal to zero:
$9(x+1)^2 - 25 = 0 \Rightarrow 9(x+1)^2 = 25$
$\Rightarrow (x+1)^2 = \frac{25}{9} \Rightarrow x + 1 = \pm\sqrt{\frac{25}{9}}$
$\Rightarrow x = -1 \pm \sqrt{\frac{25}{9}} \Rightarrow x = -1 \pm \frac{5}{3}$. So $x = \frac{2}{3}$ or $-\frac{8}{3}$

g) First complete the square of the expression:
$2x^2 - 12x + 9 = 2(x-3)^2 - 18 + 9 = 2(x-3)^2 - 9$
Now set the completed square equal to zero:
$2(x-3)^2 - 9 = 0 \Rightarrow 2(x-3)^2 = 9 \Rightarrow (x-3)^2 = \frac{9}{2}$
$\Rightarrow x - 3 = \pm\sqrt{\frac{9}{2}} \Rightarrow x = 3 \pm \sqrt{\frac{9}{2}}$
$\Rightarrow x = 3 \pm \frac{3}{\sqrt{2}} = 3 \pm \frac{3\sqrt{2}}{2}$
Here you should rationalise the denominator by multiplying the top and bottom of the fraction by $\sqrt{2}$.

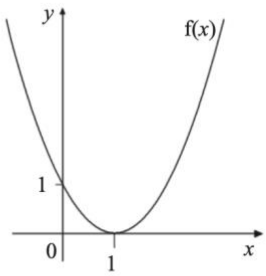
h) First divide through by 2:
$x^2 - 6x - 27 = (x-3)^2 - 9 - 27 = (x-3)^2 - 36$
Now set the completed square equal to zero:
$(x-3)^2 - 36 = 0 \Rightarrow (x-3)^2 = 36$
$\Rightarrow x - 3 = \pm 6 \Rightarrow x = 3 \pm 6$. So $x = 9$ or -3

i) Write the equation in standard quadratic form:
$5x^2 + 10x = 1 \Rightarrow 5x^2 + 10x - 1 = 0$
Then complete the square of the expression:
$5x^2 + 10x - 1 = 5(x+1)^2 - 5 - 1 = 5(x+1)^2 - 6$

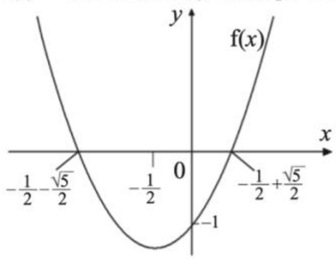
4. a) $(-3, 2)$ minimum
b) $\left(-\frac{9}{2}, \frac{45}{4}\right)$ maximum
c) $\left(\frac{11}{4}, 116\right)$ minimum

Ex 4.7 Sketching quadratic graphs

Q6 **a)** $f(x) = x^2 - 2x + 1 = (x - 1)^2$ so the function has one repeated root at $x = 1$. Letting $x = 0$ gives $f(x) = 1$ so the y-intercept is at 1. The graph is u-shaped.

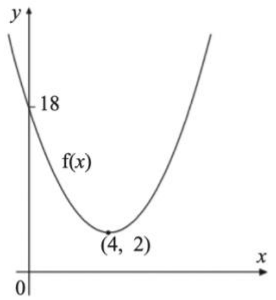


b) $f(x) = x^2 + x - 1 = \left(x + \frac{1}{2}\right)^2 - \frac{5}{4}$ and solving $f(x) = 0$ gives $x = -\frac{1}{2} \pm \frac{\sqrt{5}}{2}$ as the x-intercepts. Letting $x = 0$ we get $f(x) = -1$ so this is the y-intercept. The graph is u-shaped.

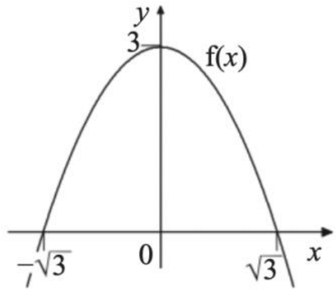


c)

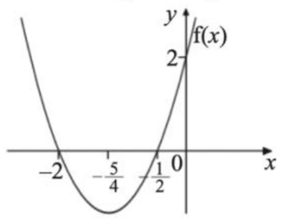
Letting $x = 0$ gives $f(x) = 18$. The graph is u-shaped but it could be one of two graphs which are u-shaped with a y-intercept of 18. To find out which, work out the vertex. It has a minimum as it is u-shaped and from completing the square, the minimum is at (4, 2).



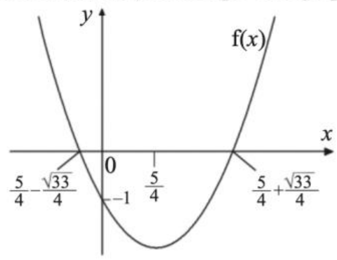
d) $f(x) = -x^2 + 3$ so setting $f(x) = 0$ gives $x = \pm\sqrt{3}$ as the x-intercepts. Letting $x = 0$ gives $f(x) = 3$ so 3 is the y-intercept. The graph is n-shaped.



e) $f(x) = 2x^2 + 5x + 2 = 2\left(x + \frac{5}{4}\right)^2 - \frac{9}{8}$ and solving $f(x) = 0$ gives $x = -\frac{5}{4} \pm \frac{3}{4} = -2$ and $-\frac{1}{2}$ as the x-intercepts. $f(0) = 2$ so this is the y-intercept. The graph is u-shaped.



f) $f(x) = 2x^2 - 5x - 1 = 2\left(x - \frac{5}{4}\right)^2 - \frac{33}{8}$ and solving $f(x) = 0$ gives $x = \frac{5}{4} \pm \frac{\sqrt{33}}{4}$ as the x-intercepts. Letting $x = 0$ we get $f(x) = -1$ so this is the y-intercept. The graph is u-shaped.



Chapter 5 Review Exercise

Q1 **a)** $7x - 4 > 2x - 42 \Rightarrow 5x > -38 \Rightarrow x > -\frac{38}{5}$

b) $12y - 3 \le 4y + 4 \Rightarrow 8y \le 7 \Rightarrow y \le \frac{7}{8}$

c) $9y - 4 \ge 17y + 2 \Rightarrow -8y \ge 6 \Rightarrow y \le -\frac{3}{4}$

d) $x + 6 < 5x - 4 \Rightarrow -4x < -10 \Rightarrow x > \frac{5}{2}$

e) $4x - 2 > x - 14 \Rightarrow 3x > -12 \Rightarrow x > -4$

f) $7 - x \le 4 - 2x \Rightarrow x \le -3$

g) $11x - 4 < 4 - 11x \Rightarrow 22x < 8 \Rightarrow x < \frac{4}{11}$

h) $1 + 10y \ge 7y - 12 \Rightarrow 3y \ge -13 \Rightarrow y \ge -\frac{13}{3}$

i) $8y - 6 \le 6 - 8y \Rightarrow 16y \le 12 \Rightarrow y \le \frac{3}{4}$

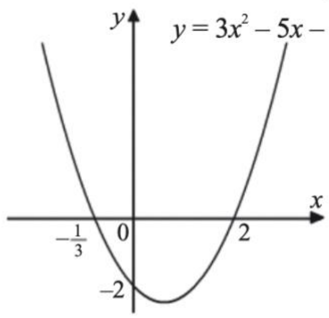
Q2 $3(2x - 5) + 2(4 - x) \ge x + 7 \Rightarrow 6x - 15 + 8 - 2x \ge x + 7$

$\Rightarrow 4x - 7 \ge x + 7 \Rightarrow 3x \ge 14 \Rightarrow x \ge \frac{14}{3}$

[3 marks available — 1 mark for expanding brackets, 1 mark for simplifying, 1 mark for correct answer]

Q3 **a)** $3x^2 - 5x - 2 = 0 \Rightarrow (3x + 1)(x - 2) = 0$

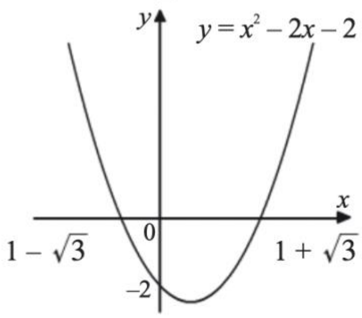
$\Rightarrow x = -\frac{1}{3}$ or $x = 2$



So: $3x^2 - 5x - 2 \le 0 \Rightarrow -\frac{1}{3} \le x \le 2$

b) $x^2 + 2x + 7 = 4x + 9 \Rightarrow x^2 - 2x - 2 = 0$

$\Rightarrow x = \frac{2 \pm \sqrt{4 + 8}}{2} \Rightarrow x = 1 \pm \sqrt{3}$

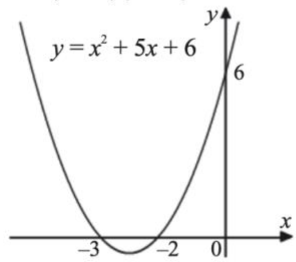


So: $x^2 + 2x + 7 > 4x + 9 \Rightarrow x^2 - 2x - 2 > 0$

$\Rightarrow x < 1 - \sqrt{3}$ or $x > 1 + \sqrt{3}$

c) $3x^2 + 7x + 4 = 2(x^2 + x - 1) \Rightarrow x^2 + 5x + 6 = 0$

$\Rightarrow (x + 3)(x + 2) = 0 \Rightarrow x = -3$ or $x = -2$

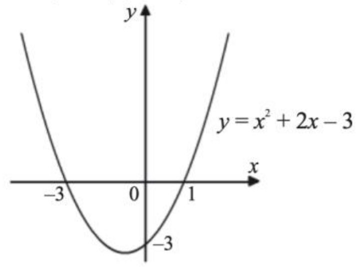


So: $3x^2 + 7x + 4 \ge 2(x^2 + x - 1)$

$\Rightarrow (x + 3)(x + 2) \ge 0 \Rightarrow x \le -3$ or $x \ge -2$

d) $x^2 + 3x - 1 = x + 2 \Rightarrow x^2 + 2x - 3 = 0$

$\Rightarrow (x + 3)(x - 1) = 0 \Rightarrow x = -3$ or $x = 1$

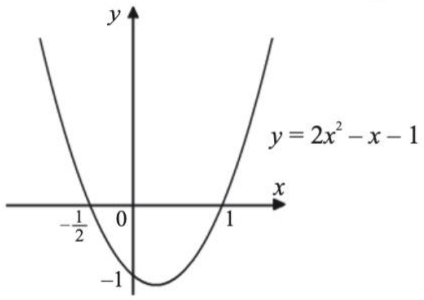


So: $x^2 + 3x - 1 \ge x + 2 \Rightarrow (x + 3)(x - 1) \ge 0$

$\Rightarrow x \le -3$ or $x \ge 1$

e) $2x^2 = x + 1 \Rightarrow 2x^2 - x - 1 = 0$

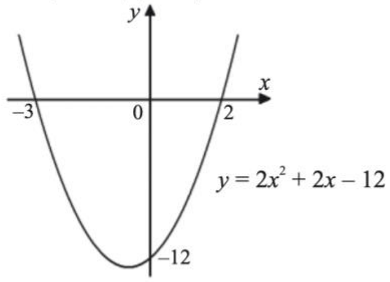
$\Rightarrow (2x + 1)(x - 1) = 0 \Rightarrow x = -\frac{1}{2}$ or $x = 1$



So: $2x^2 > x + 1 \Rightarrow (2x + 1)(x - 1) > 0 \Rightarrow x < -\frac{1}{2}$ or $x > 1$

f) $3x^2 - 12 = x^2 - 2x \Rightarrow 2x^2 + 2x - 12 = 0$

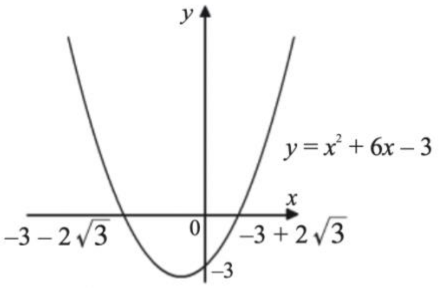
$\Rightarrow (2x - 4)(x + 3) = 0 \Rightarrow x = 2$ or $x = -3$



So: $3x^2 - 12 < x^2 - 2x \Rightarrow (2x - 4)(x + 3) < 0 \Rightarrow -3 < x < 2$

g) $3x^2 + 6x = 2x^2 + 3 \Rightarrow x^2 + 6x - 3 = 0$

$\Rightarrow x = \dfrac{-6 \pm \sqrt{36 + 12}}{2} = -3 \pm 2\sqrt{3}$



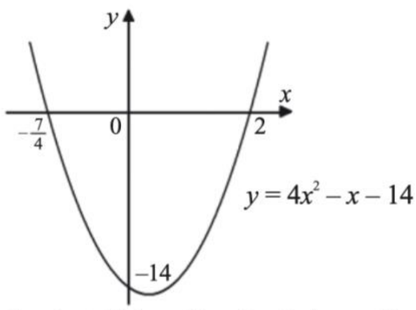
So: $3x^2 + 6x \le 2x^2 + 3 \Rightarrow x^2 + 6x - 3 \le 0$

$\Rightarrow -3 - 2\sqrt{3} \le x \le -3 + 2\sqrt{3}$

h) $(x + 2)(x - 3) = 8 - 3x^2 \Rightarrow x^2 - x - 6 = 8 - 3x^2$

$\Rightarrow 4x^2 - x - 14 = 0 \Rightarrow (4x + 7)(x - 2) = 0$

$\Rightarrow x = -\frac{7}{4}$ or $x = 2$



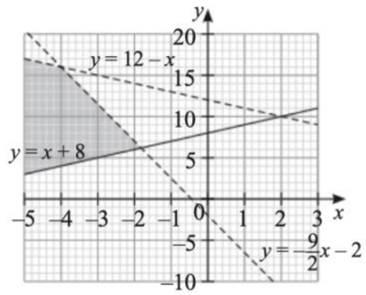
So: $(x + 2)(x - 3) \ge 8 - 3x^2 \Rightarrow (4x + 7)(x - 2) \ge 0$

$\Rightarrow x \le -\frac{7}{4}$ or $x \ge 2$

Q6 a) $8 \le y - x \Rightarrow y \ge x + 8$

$y < 12 - x$

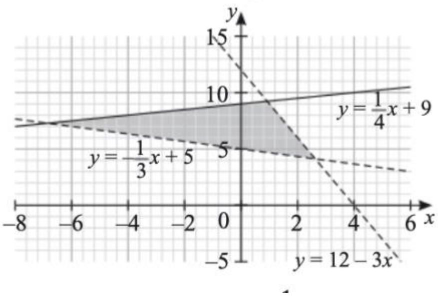
$9x + 2y < -4 \Rightarrow y < -\frac{9}{2}x - 2$



b) $x + 3y > 15 \Rightarrow y > -\frac{1}{3}x + 5$

$3x + y < 12 \Rightarrow y < 12 - 3x$

$4y \le x + 36 \Rightarrow y \le \frac{1}{4}x + 9$



Simultaneous Equations

13a) $x = -3, y = -4$

13b) $x = -\frac{1}{6}, y = -\frac{5}{12}$

13c) No solutions

13d) $x = \frac{17}{3}, y = 5$

13e) $x = 115, y = -\frac{45}{2}$

13f) $x = \frac{11}{140}, y = \frac{3}{70}$

14a) $\left(\frac{1}{4}, -\frac{13}{4}\right)$

14b) (4,5)

14c) $(-5,-2)$

14d) $\left(-\frac{157}{13}, -\frac{153}{13}\right)$

Simultaneous Equations – one linear, one quadratic (ex 5.5)

2a) $x = 4, y = 13$ or $x = -1, y = 3$

2b) $x = \frac{5}{2}, y = \frac{19}{2}$ or $x = -1, y = -1$

2c) $x = 6, y = 11$ or $x = 1, y = -4$

2d) $x = -2, y = -3$ or $x = 6, y = 1$

2f) $x = 2, y = 7$

2g) $x = -1, y = 11$ or $x = 4, y = -9$

2i) $x = 9, y = \frac{5}{4}$ or $x = \frac{10}{3}, y = -\frac{35}{9}$

4a) $(-1,13)$ and $(4,3)$

4b) $(-2,0)$ and $(2,8)$

Straight Lines

1a) $y = 3x - 7$, $3x - y - 7 = 0$

1b) $y = \frac{1}{5}x - \frac{1}{3}$, $3x - 15y - 5 = 0$

1c) $y = \frac{3}{5}x + \frac{11}{5}$, $3x - 5y + 11 = 0$

Q2 **a)** Length AB $= \sqrt{(4-(-2))^2 + (-10-4)^2}$

$= \sqrt{6^2 + (-14)^2}$

$= \sqrt{232} = 2\sqrt{58}$

[2 marks available — 1 mark for attempting to use Pythagoras' theorem on the x- and y-coordinates, 1 mark for the correct answer in surd form]

b) Gradient $m = \frac{-10-4}{4-(-2)} = \frac{-14}{6} = -\frac{7}{3}$

[2 marks available — 1 mark for change in y over change in x as a fraction, 1 mark for correct answer (or equivalent fraction)]

c) $y - y_1 = m(x - x_1) \Rightarrow y - 4 = -\frac{7}{3}(x-(-2))$

$\Rightarrow y - 4 = -\frac{7}{3}x - \frac{14}{3} \Rightarrow y = -\frac{7}{3}x - \frac{2}{3}$

$\Rightarrow 3y = -7x - 2 \Rightarrow 7x + 3y + 2 = 0$

Q3 **a)** Substituting $x = 2$ into the equation for l gives:

$y - 2(2) + 5 = 0 \Rightarrow y - 4 + 5 = 0 \Rightarrow y = -1$

So A lies on the line l. *[1 mark]*

b) $x = k \Rightarrow y - 2k + 5 = 0 \Rightarrow y = 2k - 5$ *[1 mark]*

c) The base of the triangle has length $10 - 2 = 8$.

The height of the triangle is $(2k-5) - (-1) = 2k - 4$

The area of the triangle $= \frac{1}{2} \times 8 \times (2k-4) = 32 \Rightarrow k = 6$

So C has coordinates $(6, 2(6) - 5) = (6, 7)$

So the line through B and C has gradient:

$m = \frac{7-(-1)}{6-10} = \frac{8}{-4} = -2$

$y - y_1 = m(x - x_1) \Rightarrow y - 7 = -2(x - 6) \Rightarrow y = -2x + 19$

[3 marks available — 1 mark for the correct value of k, 1 mark for the correct gradient, 1 mark for correct equation of the line]

Q4 $4x - 6y = 7 \Rightarrow y = \frac{2}{3}x - \frac{7}{6}$, so the gradient of the line is $\frac{2}{3}$.

a) $8x + 12y = 15 \Rightarrow y = -\frac{2}{3}x + \frac{5}{4}$

The gradient is $-\frac{2}{3}$ so the line is not parallel to $4x - 6y = 7$.

[2 marks available — 1 mark for calculating the gradient, 1 mark for the correct conclusion]

b) $3y - 2x = 7 \Rightarrow y = \frac{2}{3}x + \frac{7}{3}$

The gradient is $\frac{2}{3}$ so the line is parallel to $4x - 6y = 7$.

[2 marks available — 1 mark for calculating the gradient, 1 mark for the correct conclusion]

Q6 **a)** l_1 is parallel so its gradient is also $\frac{3}{2}$:

$y = mx + c \Rightarrow y = \frac{3}{2}x + c \Rightarrow 2 = \frac{3}{2}(4) + c$

$\Rightarrow c = -4 \Rightarrow y = \frac{3}{2}x - 4$

b) Rearrange the equation to get $y = 2x - 7$.

l_2 is perpendicular so the gradient is $-1 \div 2 = -\frac{1}{2}$

$y = mx + c \Rightarrow y = -\frac{1}{2}x + c \Rightarrow 1 = -\frac{1}{2}(6) + c$

$\Rightarrow c = 4 \Rightarrow y = -\frac{1}{2}x + 4$

Q7 Gradient of RS $= \frac{9-3}{1-10} = \frac{6}{-9} = -\frac{2}{3}$,

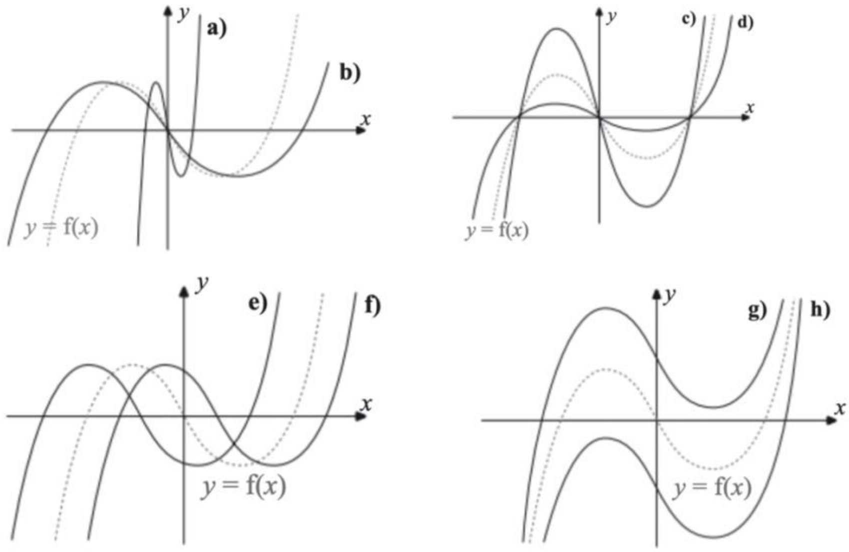
so the gradient of the perpendicular is $\frac{3}{2}$.

$y = mx + c \Rightarrow y = \frac{3}{2}x + c \Rightarrow 9 = \frac{3}{2}(1) + c$

$\Rightarrow c = \frac{15}{2} \Rightarrow y = \frac{3}{2}x + \frac{15}{2}$

Graph Transformations

Q16



Trigonometry Ex 8.1

Q6) 9.53 cm (2 s.f.)

Q7 Angle $Q = 180° - 38° - 43° = 99°$

Using the sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{PQ}{\sin 43°} = \frac{48}{\sin 99°} \Rightarrow PQ = \frac{48 \sin 43°}{\sin 99°} = 33.1$ m (3 s.f.)

Q8 Using the sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{\sin B}{14} = \frac{\sin 67°}{17} \Rightarrow \sin B = \frac{14 \times \sin 67°}{17} \Rightarrow B = 49.293...°$

So $A = 180° - 67° - 49.293...° = 63.7°$ (3 s.f.)

Q9 **a)** Using the sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{\sin x}{17} = \frac{\sin 78°}{19} \Rightarrow \sin x = \frac{17 \times \sin 78°}{19}$

$\Rightarrow x = 61.1°$ (3 s.f.)

b) Using the sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{x}{\sin 37°} = \frac{14}{\sin 102°} \Rightarrow x = \frac{14 \times \sin 37°}{\sin 102°} = 8.61$ cm (3 s.f.)

c) Using the sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{\sin x}{27} = \frac{\sin 24°}{13} \Rightarrow \sin x = \frac{27 \times \sin 24°}{13}$

$\Rightarrow x = 57.6°$ (3 s.f.)

Ex 8.4

Q3)

a) Using the cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

$\cos x = \frac{15^2 + 12^2 - 9^2}{2 \times 15 \times 12}$

$\Rightarrow x = \cos^{-1}\left(\frac{15^2 + 12^2 - 9^2}{2 \times 15 \times 12}\right) = 36.9°$ (3 s.f.)

b) Using the cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

$\cos x = \frac{14^2 + 12^2 - 21^2}{2 \times 14 \times 12}$

$\Rightarrow x = \cos^{-1}\left(\frac{14^2 + 12^2 - 21^2}{2 \times 14 \times 12}\right) = 107°$ (3 s.f.)

c) Using the cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

$\cos x = \frac{10^2 + 7^2 - 4^2}{2 \times 10 \times 7}$

$\Rightarrow x = \cos^{-1}\left(\frac{10^2 + 7^2 - 4^2}{2 \times 10 \times 7}\right) = 18.2°$ (3 s.f.)

d) Using the cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

$x^2 = 43^2 + 17^2 - (2 \times 43 \times 17 \times \cos 42°)$

$x = \sqrt{1051.5...} = 32.4$ cm (3 s.f.)

e) Using the cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

$x^2 = 56^2 + 32^2 - (2 \times 56 \times 32 \times \cos 27°)$

$x = \sqrt{966.63...} = 31.1$ mm (3 s.f.)

f) Using the cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

$x^2 = 7^2 + 13^2 - (2 \times 7 \times 13 \times \cos 54°)$

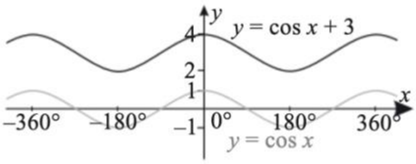
$x = \sqrt{111.02...} = 10.5$ cm (3 s.f.)

Ex 8.5

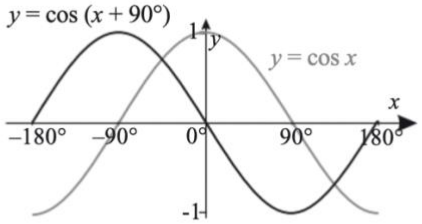
1a) 50.3 cm^2 (3 s.f.)
1b) 14.8 mm^2 (3 s.f.)
1c) 10.4 cm^2 (3 s.f.)
1d) 439 cm^2 (3 s.f.)
2) 10.6 cm^2 (3 s.f.)

Ex 8.7

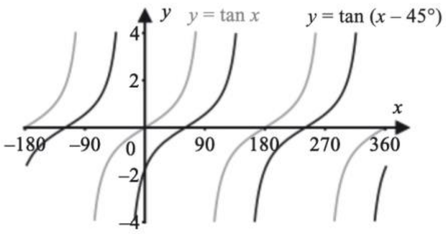
Q1



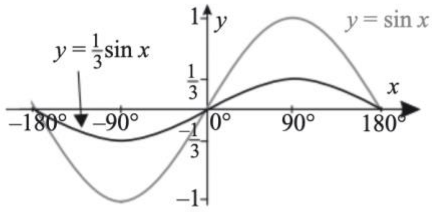
Q2



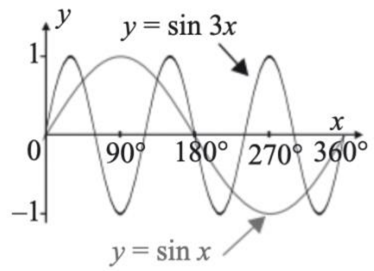
Q3



Q4

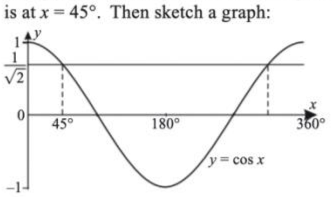


Q5



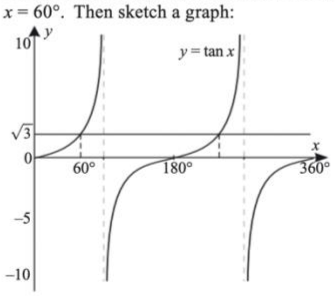
Ex 8.8

Q3 **a)** Using your knowledge of common angles, the first solution is at $x = 45°$. Then sketch a graph:



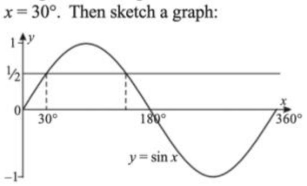
Using the symmetry of the graph, the second solution is at $360° - 45° = 315°$.

b) Using common angles, the first solution is at $x = 60°$. Then sketch a graph:



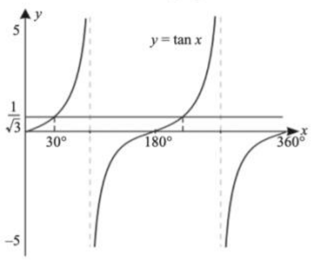
By the symmetry of the graph, the second solution is at $180° + 60° = 240°$.

c) Using common angles, the first solution is at $x = 30°$. Then sketch a graph:



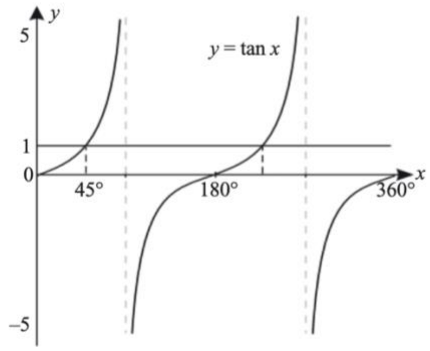
By the symmetry of the graph, the second solution is at $180° - 30° = 150°$.

d) Using common angles, the first solution is at $x = 30°$. Then sketch a graph:



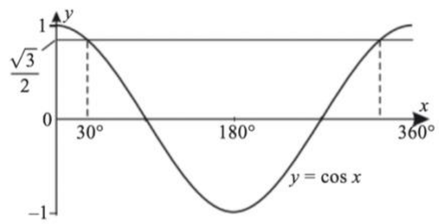
By the symmetry of the graph, the second solution is at $180° + 30° = 210°$.

e) Using common angles, the first solution is at $x = 45°$. Then sketch a graph:



By the symmetry of the graph, the second solution is at $180° + 45° = 225°$.

f) Using your knowledge of common angles, the first solution is at $x = 30°$. Then sketch a graph:



Using the symmetry of the graph, the second solution is at $360° – 30° = 330°$.

Q4 You're told that there is a solution at 44.43°. From the graph you can see that there is another solution in the given interval at $180° – 44.43° = 135.57°$ (2 d.p.).

Vectors Ex 12.1 7-10,20,21,24

Q7 **a)** $\overrightarrow{DF} = \frac{2}{3}\overrightarrow{DC}$. $\overrightarrow{DC}$ is parallel to $\overrightarrow{AB}$ and the same length because ABCD is a rectangle, so $\overrightarrow{DC} = \overrightarrow{AB} = \mathbf{b}$. So $\overrightarrow{DF} = \frac{2}{3}\mathrm{b}$.

b) $\overrightarrow{BE} = \overrightarrow{BA} + \overrightarrow{AE} = -\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AD} = -\mathrm{b} + \frac{1}{2}\mathrm{d}$

c) $\overrightarrow{EF} = \overrightarrow{ED} + \overrightarrow{DF} = \frac{1}{2}\overrightarrow{AD} + \overrightarrow{DF} = \frac{1}{2}\mathrm{d} + \frac{2}{3}\mathrm{b}$

Q8 **a)** $2\mathbf{e}$ **b)** $–2\mathbf{d}$

c) $\mathbf{d} – \mathbf{e}$ **d)** $2\mathbf{e} – \mathbf{d}$

e) $\mathbf{e} – 2\mathbf{d}$ **f)** $–\mathbf{d} – \mathbf{e}$

Q9 $\overrightarrow{JL} = \overrightarrow{JD} + \overrightarrow{DL}$

J is the midpoint of ED, so $\overrightarrow{JD} = \frac{1}{2}\overrightarrow{ED} = \frac{1}{2}\mathbf{d}$.

And L is the midpoint of DF, so $\overrightarrow{DL} = \frac{1}{2}\overrightarrow{DF}$.

$\overrightarrow{DF} = \overrightarrow{DE} + \overrightarrow{EF} = –\mathbf{d} + \mathbf{f} \Rightarrow \overrightarrow{DL} = \frac{1}{2}(\mathbf{f} – \mathbf{d})$

So, $\overrightarrow{JL} = \frac{1}{2}\mathbf{d} + \frac{1}{2}(\mathbf{f} – \mathbf{d}) = \frac{1}{2}\mathbf{f}$.

Q10 **a)** $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = –(\mathbf{p} + \mathbf{q}) + (\mathbf{p} + 2\mathbf{q}) = \mathbf{q}$ *[1 mark]*

b) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \mathbf{q} + 3\mathbf{q} = 4\mathbf{q}$

$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = \mathbf{p} + \mathbf{q} + 4\mathbf{q} = \mathbf{p} + 5\mathbf{q}$

[2 marks available — 1 mark for finding $\overrightarrow{AC}$, 1 mark for the correct answer]

Q20 $\overrightarrow{OA} = 4\mathbf{a} - 2\mathbf{b}$

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BD} + \overrightarrow{DC} = 2\mathbf{b} + 4\mathbf{a} - \mathbf{b} - \frac{5}{2}\mathbf{b} - \mathbf{a}$

$= 3\mathbf{a} - \frac{3}{2}\mathbf{b} = \frac{3}{4}\overrightarrow{OA}$

This shows that $\overrightarrow{OA}$ and $\overrightarrow{AC}$ are scalar multiples of one another, so they're parallel. Therefore, O, A and C are collinear and OAC is a straight line.

Q21 $\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB} = \mathbf{p} - (\mathbf{q} - \frac{1}{2}\mathbf{p}) = \frac{3}{2}\mathbf{p} - \mathbf{q}$

$\overrightarrow{CD} = \overrightarrow{AD} - \overrightarrow{AC} = (-5\mathbf{p} + 4\mathbf{q}) - \mathbf{p} = -6\mathbf{p} + 4\mathbf{q}$

$-6\mathbf{p} + 4\mathbf{q} = -4(\frac{3}{2}\mathbf{p} - \mathbf{q}) = -4\overrightarrow{BC}$

$\overrightarrow{BC}$ and $\overrightarrow{CD}$ are scalar multiples of one another, so they're parallel. They also meet at point C, so B, C and D are collinear.

Q24 a) $\overrightarrow{CD} = \overrightarrow{CB} + \overrightarrow{BA} + \overrightarrow{AD} = \overrightarrow{AD} - \overrightarrow{BC} - \overrightarrow{AB}$

$= \frac{3}{2}\mathbf{a} - \mathbf{a} - (\mathbf{b} - \mathbf{a}) = \frac{3}{2}\mathbf{a} - \mathbf{b}$

$\overrightarrow{BE} = \overrightarrow{BA} + \overrightarrow{AE} = \overrightarrow{AE} - \overrightarrow{AB} = (2\mathbf{a} - \mathbf{b}) - (\mathbf{b} - \mathbf{a}) = 3\mathbf{a} - 2\mathbf{b}$

$\overrightarrow{BE} = 2\overrightarrow{CD}$, so they are parallel.

b) $\overrightarrow{AO} = \overrightarrow{AB} + \overrightarrow{BO} = \overrightarrow{AB} + \frac{1}{3}\overrightarrow{BE} = (\mathbf{b} - \mathbf{a}) + \frac{1}{3}(3\mathbf{a} - 2\mathbf{b}) = \frac{1}{3}\mathbf{b}$

$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{BC} - \frac{1}{3}\overrightarrow{BE} = \mathbf{a} - \frac{1}{3}(3\mathbf{a} - 2\mathbf{b}) = \frac{2}{3}\mathbf{b}$

$\overrightarrow{AO}$ and $\overrightarrow{OC}$ are scalar multiples of one another, so they are parallel and therefore A, O and C are collinear — they lie on the same straight line.